

## Effect of soil-structure interaction on inelastic displacement ratios of degrading structures



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### ABSTRACT

This study presents an evaluation of inelastic displacement ratios for degraded structures considering soil-structure interaction (SSI). In this regard, a wide variety of effective parameters of hysteresis models and soil-structure systems are considered. Four different hysteretic models a) bilinear, b) modified Clough, c) stiffness degrading, and d) strength-stiffness degrading, are assigned to represent force-displacement response of superstructure. The supporting soil is modeled using the concept of cone models. Inelastic displacement ratios were computed for 12,000 soil-structure models with periods between 0.1 and 5 s when subjected to 19 strong ground motions recorded on NEHRP site class D. In addition, a parametric investigation is performed to evaluate the parameters that could affect nonlinear response of structures with strength-stiffness degrading hysteretic model. It is observed that generally SSI increases the inelastic displacement ratios with exception of very short period structures. Also, it is demonstrated that the soil-structure systems with stiffness degrading hysteresis model in short period range could experience larger inelastic displacement compare to those in non-degraded soil-structure systems. In particular, the SSI substantially increases inelastic displacement ratios of strength-stiffness degrading structures.

### 1. Introduction

The main objective of performance-based seismic design (PBSD) is to control the maintenance and damage level of buildings when subjected to strong ground motions with different severities. For this purpose, the values of measurable structural response parameters, such as drift and ductility are limited to acceptable values which are selected based on intended performance level. As a part of PBSD, estimation of inelastic displacement plays a key role. Some recommendations for evaluation and rehabilitation of existing structures, i.e. FEMA 356 [1], introduced an analysis procedure to compute the target inelastic displacement using equivalent inelastic single-degree-of-freedom (SDOF) system.

In this method, the target inelastic displacement can be approximated by modifying the maximum elastic displacement demand. As a known modification approach, FEMA 356 [1] introduced inelastic displacement ratio for estimating the target displacement. Inelastic displacement ratio is defined as the ratio of the maximum displacement of an inelastic single degree of freedom (SDOF) system to the maximum elastic displacement of the SDOF system with the same period and damping ratio. The relationship between maximum inelastic

displacement and maximum elastic displacement was investigated, for the first time, by Veletsos and Newmark [2]. They studied SDOF systems with elasto-plastic behavior and observed that in the low period region inelastic displacement is much greater than elastic one. However, in long period region there is no significant difference between the maximum deformation of inelastic and elastic systems which is known as equal displacement rule. A study on constant ductility inelastic displacement ratios ( $C_{\mu}$ ) demonstrated that the earthquake magnitude and site to source distance have little influence on  $C_{\mu}$  [3]. The study was conducted using 216 earthquake ground motions time history recorded on firm site condition and statistical analysis of results led to an expression for  $C_{\mu}$ .

Equations that are developed to estimate constant-ductility inelastic displacement ratios ( $C_{\mu}$ ), can be used in design process of new structures. However, the use of these equations in seismic evaluation of existing structures, may lead to an underestimation of maximum inelastic displacement [4]. Thus, Ruiz-García and Miranda [4] developed an expression to approximate mean constant strength inelastic displacement ratios ( $C_R$ ). They concluded that effects of site condition and earthquake magnitude are negligible for long period range ( $T_n > 1.0$  s). Chopra and Chintanapakdee [5] investigated inelastic displacement

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ratios of SDOF systems with bilinear behavior and developed equations for  $C_R$  and  $C_\mu$ . Ruiz-Garcia and Miranda [6] conducted a comprehensive statistical study on inelastic displacement ratios of structures located on soft soil site condition and proposed an equation for estimating inelastic displacement ratios. They evaluated the influences of earthquake magnitude, site to source distance, post yield stiffness, stiffness and strength degradation on constant strength inelastic displacement ratios. Ruiz-Garcia and Miranda [6] concluded that the stiffness and strength degradation have significant effects on inelastic displacement of structures built on soft soil. Also, the results showed that the combination of stiffness and strength degradation can increase inelastic displacement of structures with periods less than half of the predominant period of the ground motion.

In all the aforementioned studies the effects of soil-structure interaction (SSI) on nonlinear response of SDOF systems were ignored, even though it is known that SSI affects the linear and nonlinear response of structures. Based on the studies conducted in early 1970s, SSI effects on elastic systems could be divided into two parts. First, period of soil-structure system is greater than the fixed one and second, considering SSI increases the effective damping ratio of soil-structure system because of radiation and material damping of soil beneath the structure [7]. Thus, regulation codes suggest an equivalent fixed base system with modified fundamental period and damping ratio to include the SSI effects [8,9].

Avilés and Pérez-Rocha [10] evaluated the effects of soil-structure interaction on inelastic systems response and proposed an equivalent fixed base nonlinear system which is defined by effective ductility, period and damping ratio. The further study is conducted to investigate the influences of SSI on strength reduction factor and inelastic displacement ratios of elasto-plastic SDOF systems [11]. They used equivalent fixed-base nonlinear system in order to adjust the equation that had been proposed by Ordaz and Perez-Rocha [12], to consider the effects of SSI on strength reduction factor. Response data for 64 ground motions recorded on different site conditions demonstrated, that the SSI effects on  $C_\mu$  for structures built on soft and very soft soil conditions should be considered, especially in short period region [13]. The results of this study are used to develop equations for estimation of  $C_\mu$  regarding SSI effects and site conditions. Several other research efforts have focused on the evaluation of SSI effects on inelastic displacement ratios [14–16]. However, it should be noted that the results of these studies were restricted to the structural systems with bilinear hysteretic behavior.

In order to evaluate the influences of SSI on constant strength inelastic displacement ratios ( $C_R$ ) of structures with stiffness degrading, Aydemir [17] conducted a study and concluded that the mean inelastic displacement ratios for degrading systems are greater than the corresponding ones of non-degrading systems up to period of nearly 1.0 s. In this study equivalent nonlinear fixed-base systems with modified Clough hysteresis model were used and the results led to an equation for estimating the inelastic displacement ratios of stiffness degrading soil-structure systems.

Based on new seismic provisions, it is assumed that the structures behave inelastically during sever earthquakes and will experience large inelastic deformation without losing strength, considerably [8,18]. However, it is observed that during strong ground motion, strength and stiffness degradation may occurred in structural components [19]. Deterioration in structural components can significantly increase lateral displacement and may lead to global collapse of structures. In several studies the collapse of structures are considered as the association of P-Delta effects and structural components deterioration [20]. Most of the previous studies that investigated the effects of strength-stiffness degradation on inelastic displacement demands were limited to fixed-base structures and the effects of soil-structure interaction were ignored. Aydemir [17] considered both foundation flexibility and stiffness degrading effects on inelastic displacement ratios. In practice most of structures exhibit stiffness degradation at unloading and reloading

branches, whereas the unloading stiffness in modified Clough hysteresis model which is used by Aydemir [17], is kept equal to the initial elastic stiffness.

Recently there has been a renewed interest on the effects of SSI on inelastic response of structures and several studies demonstrated that soil beneath the structures could have significant effects on seismic demands of structures [21–25]. However, the effects of soil-structures interaction on seismic demands of degraded systems have not yet been well addressed and it is, thus, necessary to clarify the influences of SSI on these type of systems. In this study the effects of foundation flexibility on degraded super-structures are investigated. To model degraded super-structure, four different hysteresis behaviors were considered: a) Bilinear, b) modified Clough, c) stiffness degrading and d) stiffness-strength degrading. A parametric study is performed by using a simplified soil-structure interaction model with a wide range of effective parameters. Here, in this paper the results of a comprehensive statistical study on displacement demand and inelastic displacement ratios of degraded structural systems by considering soil-structure interaction are presented.

## 2. Soil-structure model

Seismic responses of structure to strong ground motions are affected by supporting soil behavior which is called soil-structure interaction (SSI). In general, these effects could be divided into two parts, which are known as; inertial – and kinematic effects [26]. In this investigation a simplified SDOF soil-structure model is applied to consider the inertial interaction phenomenon on super-structure seismic demands (Fig. 1). This model is capable enough to simulate the effects of SSI in MDOF systems. For this purpose, effective mass  $M^*$  and effective height  $H^*$  of equivalent SDOF system could be obtained from fundamental mode properties of MDOF system as follow [27]:

$$M^* = \frac{\left(\sum_{j=1}^n m_j \phi_{j1}\right)^2}{\sum_{j=1}^n m_j \phi_{j1}^2} \quad H^* = \frac{\sum_{j=1}^n h_j m_j \phi_{j1}}{\sum_{j=1}^n m_j \phi_{j1}} \quad (1)$$

where  $m_j$  is the mass of the  $j$ th storey;  $h_j$  is the height from the base level to level  $j$ ; and  $\phi_{j1}$  is the amplitude at  $j$ th storey of the first mode.

In practice, various approaches are used to model supporting soil in SSI problems, such as finite element and finite difference approaches. However, in this study lumped-parameter model (Cone model) is used, because of its simplicity and sufficient accuracy [28] (Fig. 2). In cone model, the soil beneath the structure is modeled as a homogenous half-space and the foundation is assumed to be rigid with circular shape

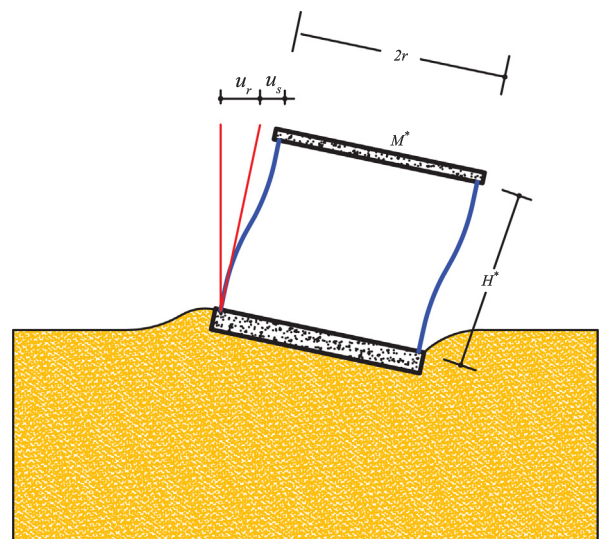


Fig. 1. SDOF soil-structure model used in this study.

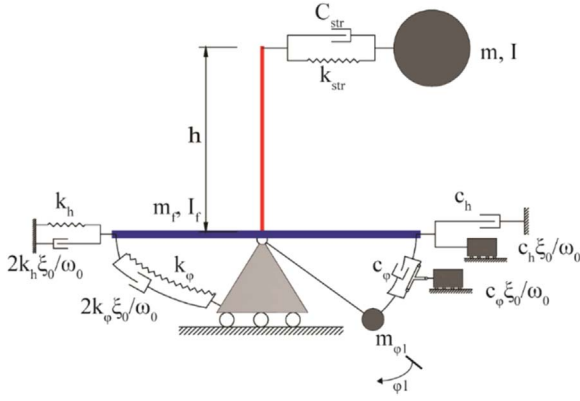


Fig. 2. Soil-structure SDOF system (Cone model).

[29]. As shown in Fig. 2, the supporting soil is substituted with a 3 DOF spring and dashpot system. A translational and a rotational spring are used to represent the sway and rocking degree-of-freedom. Also, viscous dampers are used to simulate energy dissipation of the supporting soil due to radiation and material damping. An internal DOF  $\varphi_1$ , is introduced to consider frequency dependency of the rotational spring and dashpot [29]. The adopted coefficients of the soil-foundation model are:

$$k_h = \frac{8\rho V_s^2 r}{2-\nu} \quad c_h = \pi\rho V_s r^2 \quad (2)$$

$$k_\varphi = \frac{8\rho V_s^2 r^3}{3(1-\nu)} \quad c_\varphi = \frac{\pi\rho V_p r^4}{4} \quad (3)$$

$$M_{\varphi_1} = \frac{9\rho\pi^2 r^5 (1-\nu)}{32} \left(\frac{V_p}{V_s}\right)^2 \quad (4)$$

In Eqs. (2)–(4),  $k_h$ ,  $c_h$ ,  $k_\varphi$  and  $c_\varphi$  are sway stiffness, sway viscous damping, rocking stiffness and rocking damping, respectively.  $\rho$ ,  $\nu$ ,  $V_p$  and  $V_s$  are respectively the specific mass, Poisson's ratio and the dilatational and shear wave velocities of soil and  $r$  is the radius of the equivalent circular foundation. Also, to modify the effect of incompressibility of soil, an additional mass moment of inertia  $\Delta M_\varphi$  equal to  $0.3\pi(\nu-1/3)\rho r^2$  is added to  $I_f$  for  $\nu$  greater than 0.3 [29].

The super-structure is modeled as a nonlinear SDOF system with the same damping ratio and period of vibration as those of fixed-base structure. In this study, bilinear, modified Clough, stiffness degrading and strength-stiffness degrading models are used to represent the hysteretic response of super-structure. Also, the parameters  $m$  and  $h$  are used to describe the effective mass and effective height of structure, respectively.

Seismic response of soil-structure system to earthquake ground motion depends on dynamic properties of the soil and super-structure as well as specific excitation. These effective parameters can be presented by basic non-dimensional parameters as follow [30]:

- A dimensionless frequency as an index for the structure to soil stiffness ratio defined as:

$$a_0 = \frac{\omega_{fix} h}{V_s} \quad (5)$$

where  $\omega_{fix}$  is the circular frequency of the fixed base structure. The practical range of this index for conventional building type structures can change from zero for the fixed-base structures up to 3 for sever SSI effect [30].

- Aspect ratio of the building  $h/r$ , where  $h$  is the height of the structure and  $r$  is the radius of the equivalent circular foundation.
- Structure-to-soil mass ratio index defined as:

$$\bar{m} = \frac{m}{\rho r^2 h} \quad (6)$$

- Foundation to structure mass ratio index  $m_f/m$ .
- Poisson's ratio of soil  $\nu$ .

The first two parameters, with vast range of variations are the most effective indices, which affect the response of SSI systems, and, thus, are usually introduced as key parameters. The structure-to-soil mass ratio varies between 0.4 and 0.6 for ordinary building type structures and is considered 0.5 in present study [30]. The foundation-to-structure mass ratio is assigned 0.1, and the Poisson's ratio to be 0.4. The damping ratio of soil material is considered to be 5% of the critical damping at the effective period of SSI system. Damping ratio of the superstructure is assumed to be 5% of the critical damping.

### 3. Selected earthquake ground motions

In this investigation an ensemble of 19 earthquake acceleration time histories were selected from strong ground motion database of the Pacific Earthquake Engineering Center (PEER, <http://ngawest2.berkeley.edu/>). The selected ensemble is categorized by NEHRP site class D [18]. These ground motions were recorded during 9 earthquakes with magnitude ranging from 6.5 to 7.6 at distance ranging from 10 to 28 km (closest distance to the ruptured fault area).

All the selected strong ground motions have the following characteristics: (i) earthquakes with magnitude greater than 6.5; (ii) Distance from source to site > 10 km; (iii) records in which at least one of the two horizontal components had a peak ground acceleration > 0.2 g and peak ground velocity > 15 cm/sec; (iv) selected records are not identified as pulse like ground motion in PEER strong ground motion database. The main characteristics of the selected ground motions are listed in Table 1. For each record, the horizontal component with larger peak ground velocity is defined as the “strong” component and the other one is denoted as “weak” component. In this study, results were obtained based on the strong-component ensemble. Elastic response spectrums of strong-component ensemble with their mean values are presented in Fig. 3.

### 4. Load displacement hysteretic models

In this study, the main objective is to investigate the effects of soil-structure interaction on inelastic displacement demands of degraded structures. For this regard, four different load-deformation hysteretic models are considered: a) bilinear (BL); b) modified Clough (CL); c) stiffness degrading (SD) and d) modified Ibarra-Medina-Krawinkler bilinear (MIB).

The BL hysteresis model is used in this study to represent structures without stiffness and strength degradation, thus, its response serves as a baseline. Only three parameters are needed to characterize BL model, the initial stiffness ( $k$ ), yielding strength ( $F_y$ ) and strain-hardening ratio ( $\alpha$ ).

When concrete structures are subjected to reversed cyclic loading, they will exhibit stiffness degradation. The modified Clough hysteresis model is used to modeling this behavior in concrete structures. At first, this model had been proposed by Clough and Johnston [31] and then were modified by Mahin and Bertero [32]. This model has a bilinear envelope, however after the initial yielding, further loading branches are directed towards the furthest unloading point in the direction of loading [33]. The unloading stiffness in this model is kept equal to the initial elastic stiffness.

Stiffness-degrading model is used to represent the global behavior of structures exhibiting the stiffness degradation at unloading and re-loading branches. Even in well detailed structures a reduction in lateral stiffness under unloading and reloading has been observed during cyclic loading. Thus, the stiffness degrading model is calibrated to simulate this behavior.

**Table 1**  
Earthquake ground motions recorded on NEHRP site class D used in this study.

EQ index	Record ID	Event <sup>a</sup>	Mag.	Station name	Preferred NEHRP based on Vs30	ClstD (km) <sup>b</sup>	Strong			Weak		
							Ag (g)	Vg (cm/s)	Dg (cm)	Ag (g)	Vg (cm/s)	Dg (cm)
1	RSN68	1	6.61	LA - Hollywood Stor FF	D	22.8	0.2	21.7	15.9	0.2	16.9	12.9
2	RSN162	2	6.53	Calexico Fire Station	D	10.5	0.3	22.5	9.9	0.2	18.7	15.9
3	RSN169	2	6.53	Delta	D	22.0	0.3	33.0	20.2	0.2	26.3	14.7
4	RSN174	2	6.53	El Centro Array #11	D	12.6	0.4	44.6	21.3	0.4	36.0	25.1
5	RSN721	3	6.54	El Centro Imp. Co. Cent	D	18.2	0.4	48.1	19.3	0.3	41.8	21.9
6	RSN728	3	6.54	Westmorland Fire Sta	D	13.0	0.2	32.3	22.3	0.2	23.5	15.0
7	RSN752	4	6.93	Capitola	D	15.2	0.5	38.0	7.1	0.4	29.6	4.9
8	RSN776	4	6.93	Hollister - South & Pine	D	27.9	0.4	63.0	32.3	0.2	30.9	19.7
9	RSN777	4	6.93	Hollister City Hall	D	27.6	0.2	45.5	28.5	0.2	38.9	19.4
10	RSN778	4	6.93	Hollister Differential Array	D	24.8	0.3	44.2	19.7	0.3	35.8	14.6
11	RSN953	5	6.69	Beverly Hills - 14145 Mulhol	D	17.2	0.5	66.7	12.2	0.4	59.3	15.5
12	RSN960	5	6.69	Canyon Country - W Lost Cany	D	12.4	0.4	44.4	11.3	0.5	41.1	14.6
13	RSN1003	5	6.69	LA - Saturn St	D	27.0	0.4	41.6	5.0	0.5	37.2	4.4
14	RSN1077	5	6.69	Santa Monica City Hall	D	26.5	0.9	41.6	15.2	0.4	25.0	7.4
15	RSN1107	6	6.90	Kakogawa	D	22.5	0.3	26.9	8.8	0.2	20.8	6.4
16	RSN1116	6	6.90	Shin-Osaka	D	19.2	0.2	31.3	8.4	0.2	21.8	9.7
17	RSN1158	7	7.51	Duzce	D	15.4	0.3	58.9	44.1	0.4	55.7	25.0
18	RSN1203	8	7.62	CHY036	D	16.0	0.2	44.8	34.0	0.3	41.7	19.5
19	RSN3749	9	7.01	Fortuna Fire Station	D	20.4	0.3	38.1	16.7	0.3	33.9	20.9

<sup>a</sup> (1) San Fernando; (2) Imperial Valley-06; (3) Superstition Hills-02; (4) Loma Prieta; (5) Northridge-01; (6) Kobe, Japan; (7) Kocaeli, Turkey; (8) Chi-Chi, Taiwan; (9) Cape Mendocino.

<sup>b</sup> Closest distance from the recording site to the ruptured fault area.

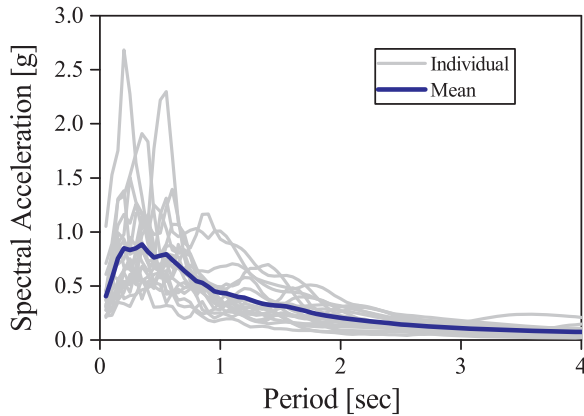


Fig. 3. Elastic response spectra of selected ground motions.

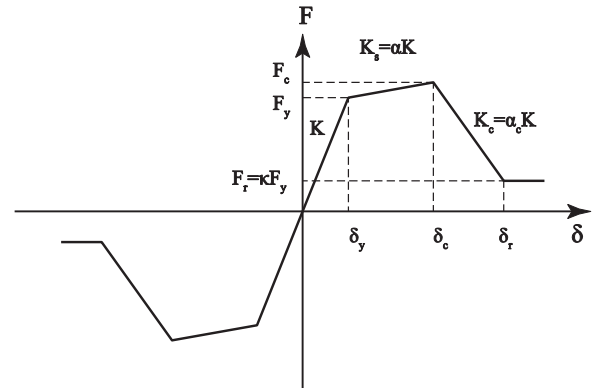


Fig. 4. Modified Ibarra-Medina-Krawinkler hysteresis model.

During strong earthquake ground motion, some structures experience a reduction in lateral strength in conjunction with stiffness degradation, especially in non-ductile structures. Modified Ibarra-Medina-Krawinkler-bilinear model (MIB) is used to simulate the hysteretic response of structures which exhibit both stiffness and strength degradation [20]. Modified Ibarra-Medina-Krawinkler-bilinear model (MIB) is based on the standard bilinear hysteretic rules with kinematic strain hardening. This model consists of four branches: (i) elastic portion, (ii) plastic portion, (iii) softening branch and (iv) residual strength portion (Fig. 4). A graphical representation of the selected hysteresis models and their corresponding parameters are shown in Fig. 5. It should be noted that the parameters of MIB model are chosen appropriate to model in-cycle degradation [19].

**5. Response variable**

Inelastic displacement ratio,  $C_R$ , is defined as ratio of the maximum inelastic displacement demand ( $S_{di}$ ) to the maximum elastic displacement ( $S_{de}$ ) on system with the same mass and initial stiffness when subjected to a given earthquake ground motion [6].

$$C_R = \frac{S_{di}}{S_{de}} \tag{7}$$

The peak inelastic displacement in Eq. (7) is computed in systems with constant strength reduction factor,  $R$ , which is defined as:

$$R = \frac{F_y}{F_e} \tag{8}$$

where  $F_e$  is the strength demand to maintain the system elastic and  $F_y$  is the yield strength of the system. Constant strength spectrum for each record and selected  $R$  is computed as follow:

1. Select a ground motion.
2. Select and fix the damping ratio  $\zeta$  for which the spectrum is to be obtained.
3. Select the target fixed-base period of vibration ( $T_{fix}$ ).
4. Determine the response of linear system with  $T_{fix}$  and  $\zeta$  equal to the selected values. The peak elastic force  $F_e$  is determined from response of linear system.
5. Determine the displacement response of an inelastic system with the same  $T_{fix}$  and  $\zeta$  and yield strength  $F_y = F_e/R$ .
6. Repeat steps 1–5 for all of the interested periods.



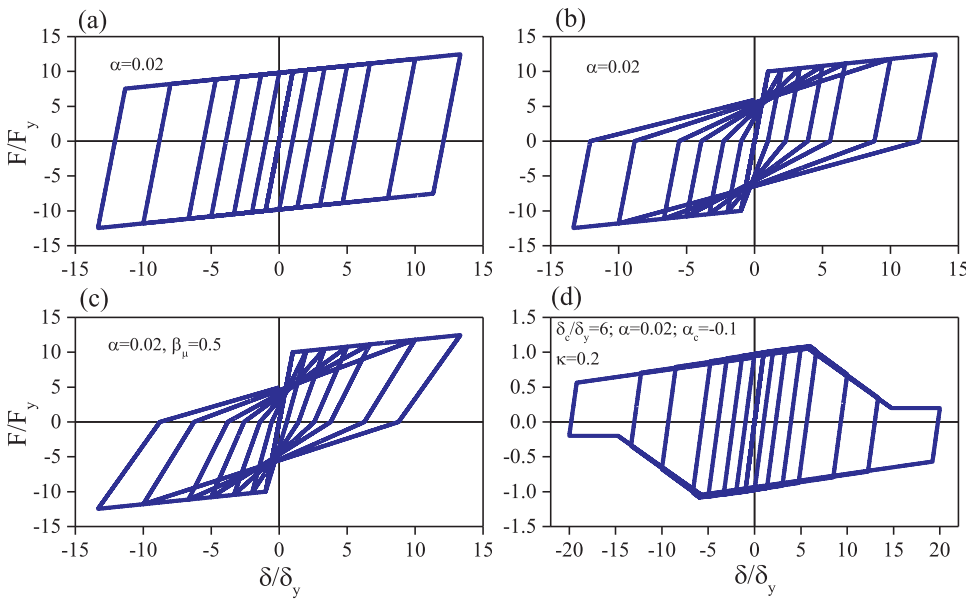


Fig. 5. Load-deformation hysteretic model used in this study: a) Bilinear; b) Modified Clough; c) stiffness degrading; d) Modified Ibarra-Medina-Krawinkler Bilinear.

### 6. Result of statistical study

Extensive nonlinear dynamic analyses were performed to evaluate the effects of SSI on inelastic displacement ratios of degrading and non-degrading super-structures. The results were obtained for soil-structure systems having 35 periods of vibration between 0.1 and 5 s, 4 hysteresis behaviors, 3 aspect ratios ( $h/r = 1, 3, 5$ ), 3 non-dimensional frequencies ( $a_0 = 1, 2, 3$ ) and 5 levels of strength reduction factors ( $R = 1.5, 2, 4, 6, 8$ ) subjected 19 strong ground motions. As a part of this study, the effects of strength reduction level, foundation flexibility, aspect ratio and hysteretic behavior on inelastic displacement ratios are primarily investigated. Afterwards, a large variety of soil-structure systems with MI (Modified Ibarra-Medina-Krawinkler) hysteretic behavior are used to evaluate the parameters that could affect nonlinear response of strength-stiffness degrading systems. The nonlinear dynamic analyses were conducted by OpenSees software developed by the Pacific Earthquake Engineering Research Center (PEER) [34].

#### 6.1. Evaluation of inelastic displacement ratios

Variations of mean constant-strength inelastic displacement ratios of fixed-base systems as a function of periods for each hysteretic behavior are shown in Fig. 6. As can be seen, in short period region the maximum displacement of nonlinear systems are significantly greater than of the elastic ones while with increase in the period of vibration, inelastic displacement ratios tend to reach unity. The spectral region where the peak inelastic displacement demands are approximately equal to the peak elastic displacement demands, is known as equal displacement region. As shown in Fig. 6, the threshold period that divides these two spectral regions increases with decreasing the strength demand (i.e., increasing the value of strength reduction factor). Also, it is observed that strength-stiffness degradation has significant effects on equal displacement region and shifts it to a longer period range.

Mean inelastic displacement ratios of soil-structure systems as a function of fixed-base periods of vibration, corresponding to each of the previously mentioned hysteresis behaviors are shown in Figs. 7–9. It can be seen that, in all the hysteresis models inelastic displacement ratios of soil-structure systems show a similar trend, which are primarily dependent on the non-dimensional frequency ( $a_0$ ), slenderness ratio ( $h/r$ ), and strength reduction factor ( $R$ ). As shown, inelastic displacement ratios spectra consist of two parts. First in short period region, the  $C_R$  value increases as the period of vibration decreases. In this region, inelastic displacement ratios spectra of soil-structure systems with BL, CL and SD hysteresis behaviors are strongly

dependent on strength reduction factor ( $R$ ) such that any increment in  $R$  amplifies the  $C_R$  values of short period structures.

Second, in long period region the  $C_R$  values tend to reach a constant value. The limiting value of the second part of the spectra and the corresponding period region is not the same for various values of  $a_0$  and  $h/r$ . Fig. 7 shows that the  $C_R$  values of soil-structure systems with BL, CL and SD hysteresis behaviors in long period region are slightly greater than one. However, an increment in non-dimensional frequency ( $a_0$ ) causes an increase in  $C_R$  values of these structures (Fig. 9). In addition, slenderizing the long period structures lead to higher values of  $C_R$ , especially for greater values of strength reduction factor.

For a specific value of non-dimensional frequency, increasing the aspect ratio ( $h/r$ ) usually has two remarkable effects. First, any increase in  $h/r$  value will be accompanied by the elongation the period of soil-structure system and second, slenderizing the structure leads to a decrease in radiation damping. The combination of these two effects causes the displacement of the soil-structure system having longer period of vibration to be decreased. However, the influence of slenderizing the long period structures with nonlinear behavior is lower than the elastic one (Fig. 10). Therefore, the  $C_R$  values of long period structures will be intensified by increasing the aspect ratio.

According to ASCE-41-13 [35],  $C_1$  coefficient is introduced as a modification factor to relate expected maximum inelastic displacement to displacements calculated for linear elastic response. This coefficient is used to calculate the target displacement in the nonlinear static procedure which is defined as:

$$C_1 = 1 + \frac{R - 1}{aT_e^2} \tag{9}$$

where  $a$  is the site class factor and  $T_e$  is the effective fundamental period of the structure. For period less than 0.2 s,  $C_1$  need not be taken greater than the value at  $T = 0.2$  s. For periods greater than 1 s,  $C_1 = 1$ .

Eq. (9) focused on the site effect and obviously, using Eq. (9) which has been originally suggested for fixed-base structures, would not be suitable for computing  $C_R$  for soil-structure systems. Fig. 11 shows the comparison of  $C_R$  with  $C_1$  coefficient computed from Eq. (9) for fixed-base and soil-structure systems with bilinear hysteresis model. As expected the  $C_1$  coefficient proposed by ASCE-41-13 [35] underestimate inelastic displacement ratios of soil-structure systems, especially when the soil-structure interaction effects are substantial (i.e.,  $a_0 = 3, h/r = 5$ ). As mentioned above, in flexible-base long-period structures the  $C_R$  will not descend toward unity but takes larger values depending on SSI key parameters and structural yielding level.

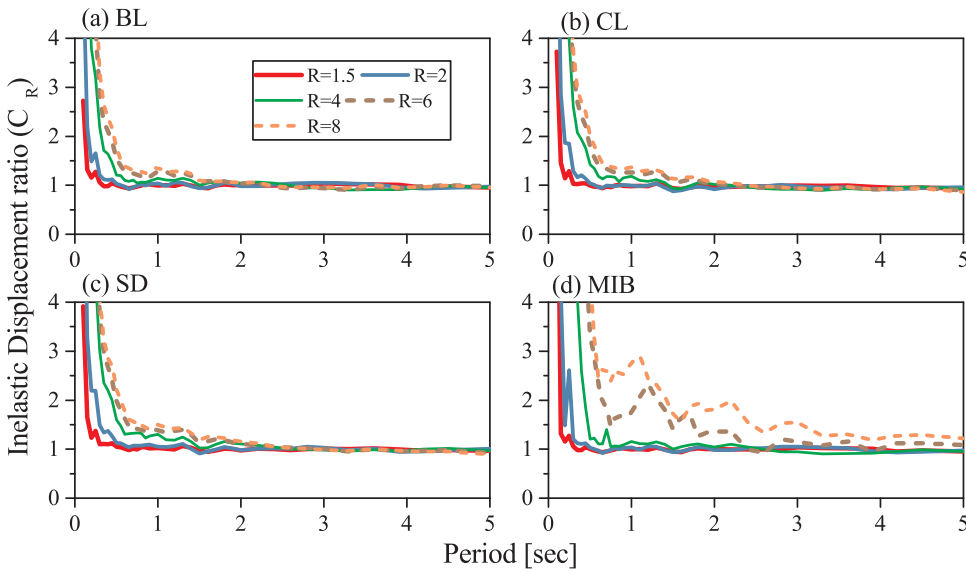


Fig. 6. Inelastic displacement ratios of fixed base systems with considered hysteresis models.

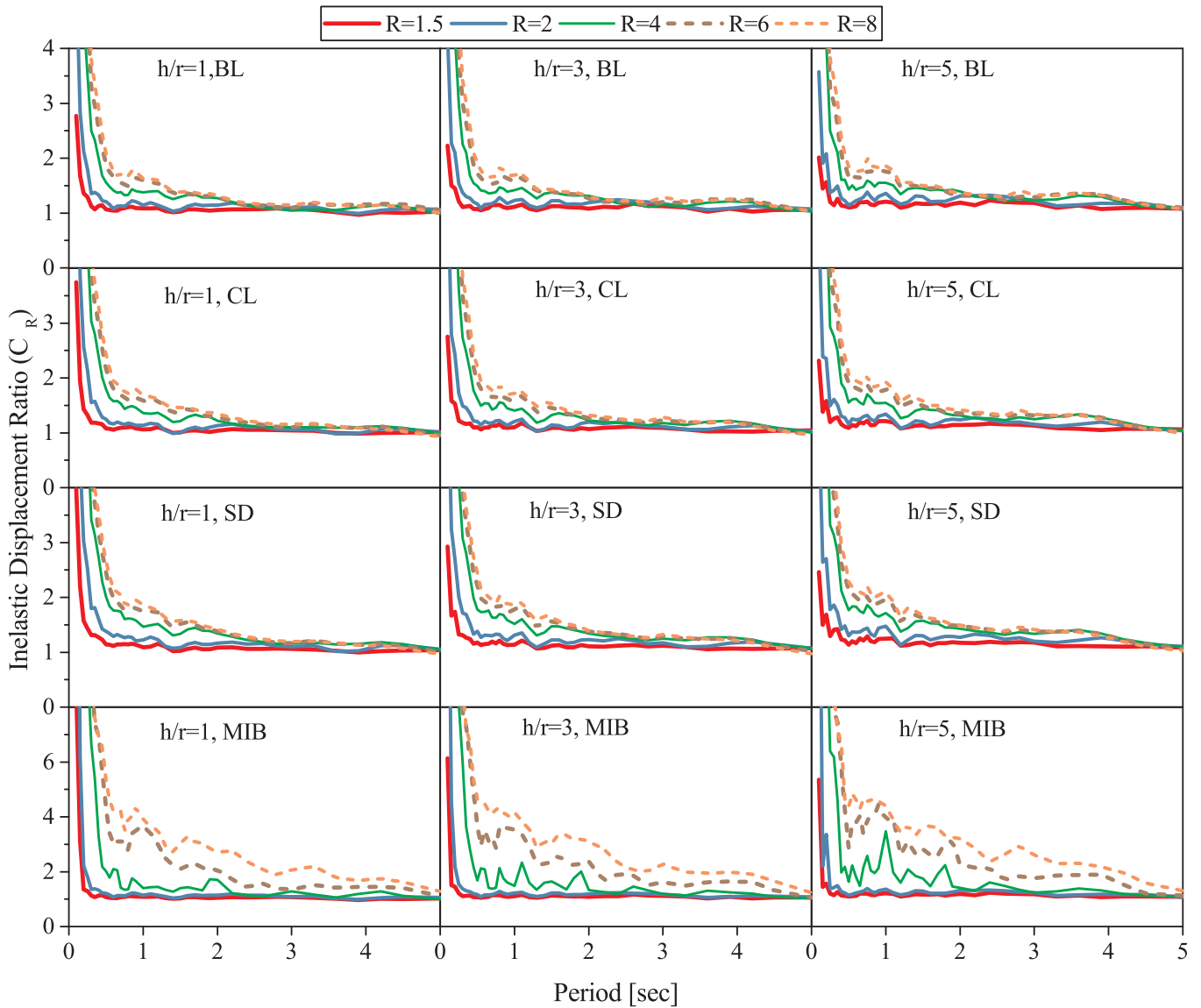


Fig. 7. Inelastic displacement ratios of soil-structure systems with considered hysteresis models for  $a_0 = 1$ .

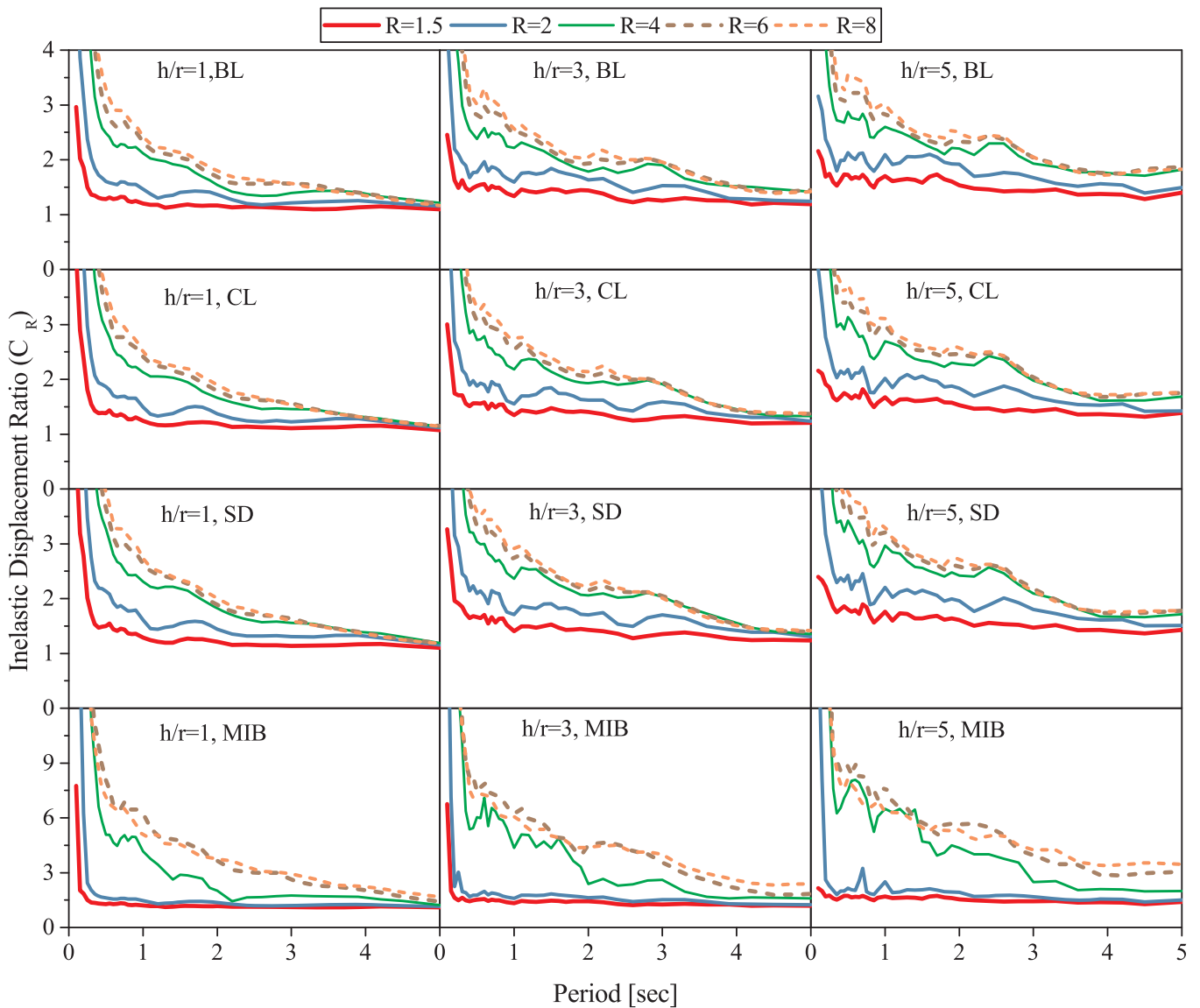


Fig. 8. Inelastic displacement ratios of soil-structure systems with considered hysteresis models for  $a_0 = 2$ .

In general, inelastic displacement of structure during earthquake ground motion increases as the lateral yielding strength of structure decreases (i.e., strength reduction factor increases) [36]. However, as shown in Fig. 9, for the cases of MIB model inelastic displacement demands descend for strength reduction factors greater than 4. In order to explain this phenomenon, the displacement time history and hysteretic response of soil-structure systems with MIB hysteresis model for  $h/r = 1$  and  $a_0 = 3$  are depicted in Fig. 12. It can be seen that for  $R$  values lower than 4, the peak inelastic displacement value increases as the lateral yielding strength of structure reduces, whereas for strength reduction greater than 4 peak inelastic displacement decreases. It is observed that for structures with  $R$  values greater than 4, hysteretic response reaches the residual branch of hysteretic model. Therefore, in MIB model, variations of  $C_R$  values, versus the strength reduction factor depend on the hysteretic behavior of the structure and an increment in  $R$  value may lead to a reduction in the inelastic displacement when the structures reach its residual state.

### 6.2. Dispersion of inelastic displacement ratio

The coefficients of variation (COV) is a measure of dispersion that describes the amount of variability relative to the mean. By definition, the COV is unit-less and could be used instead of standard deviation to

compare the spread of data set having different means. Fig. 13 shows the COV of inelastic displacement ratios corresponding to all the considered hysteresis behaviors. In Fig. 12, columns (a), (b) and (c) show the effects of strength reduction factor, non-dimensional frequency and aspect ratio on COV, respectively. The results are obtained from Fig. 13, could be summarized as: a) an increment in  $R$  will increase the dispersion (COV), with exception of the systems with MIB hysteresis behavior.

As mentioned before, in MIB model when the structure reaches residual branch of hysteretic model, inelastic displacement of structure may reduce with increasing in  $R$  value. In Fig. 13a, the inelastic displacement ratios of structures with MIB hysteresis behavior descend for  $R$  greater than 4, leading to a reduction in COV of  $C_R$ .

b) Dispersion of  $C_R$  reduces as the non-dimensional frequency ( $a_0$ ) increases, especially in systems with MIB hysteresis behavior. c) Slenderizing the long period soil-structure systems causes a reduction in the COV of inelastic displacement ratios.

### 6.3. Effect of foundation flexibility

To examine the effects of soil beneath the structures on inelastic displacement ratios of soil-structure systems, the ratios of  $C_R$  values for flexible base structures to fixed-base ones were computed ( $\gamma$ ).

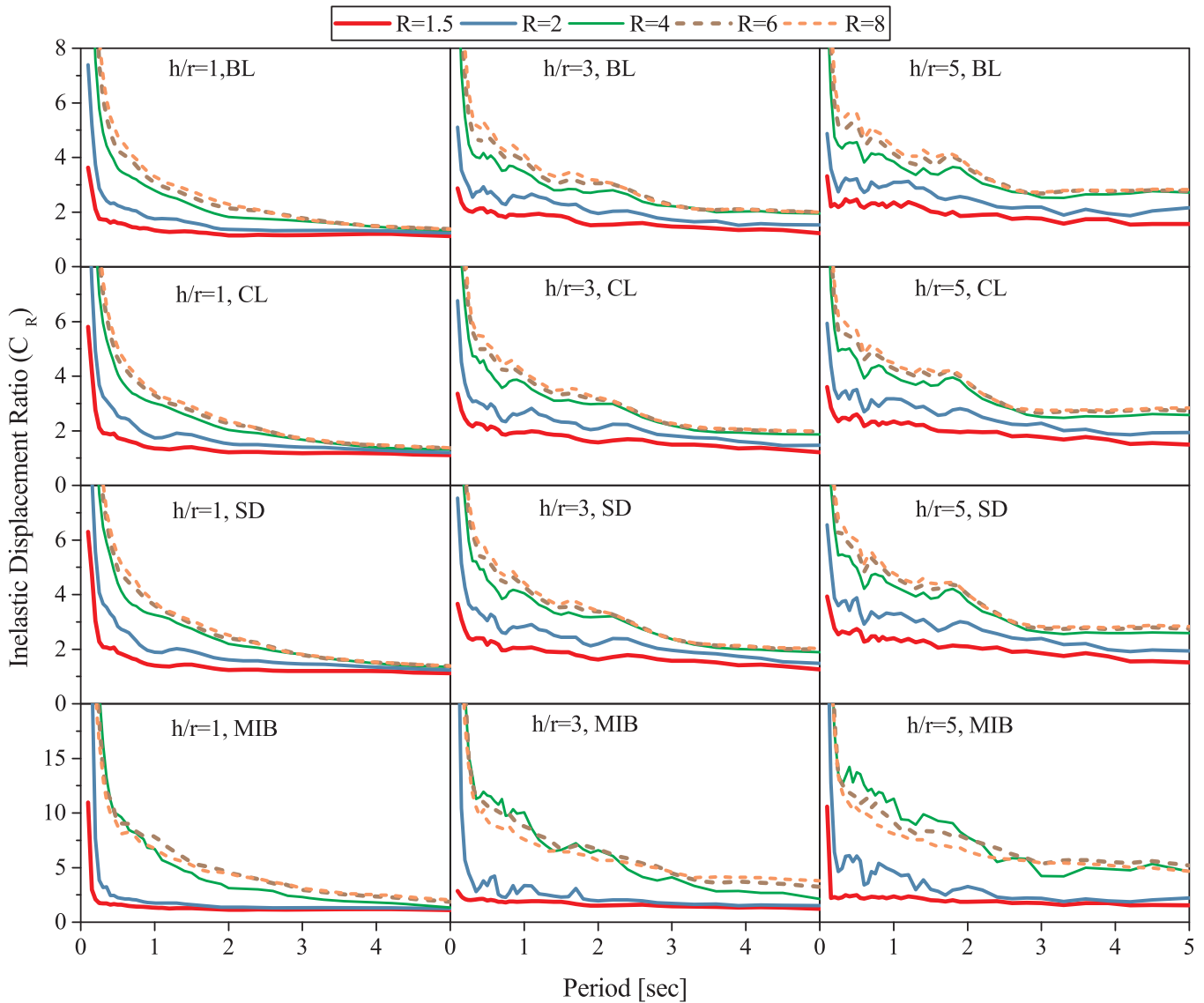


Fig. 9. Inelastic displacement ratios of soil-structure systems with considered hysteresis models for  $a_0 = 3$ .

$$\gamma = \frac{C_{R,SSI}}{C_{R,fix}} \quad (10)$$

The ratio was computed for various values of fixed-base periods and strength reduction factors as well as for different hysteresis behaviors subjected to the selected earthquake ground motions listed in Table 1. Fig. 14a shows the mean  $\gamma$  values for  $h/r = 3$ ,  $a_0 = 2$ . It can be seen

that, with exception of structures with very short period of vibration, the  $C_R$  values of soil-structure systems are greater than those of the corresponding fixed base ones.

For the cases of BL, CL and SD hysteresis behaviors the  $\gamma$  value increases with increase in strength reduction factor. This conclusion is not valid for the structures with MIB hysteresis behavior, because, as

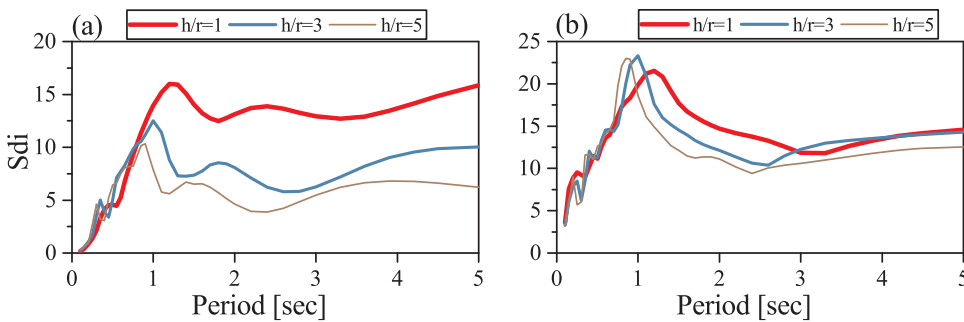


Fig. 10. Inelastic displacement spectra for record RSN953,  $a_0 = 2$ : a)  $R = 1$  and b)  $R = 8$ .



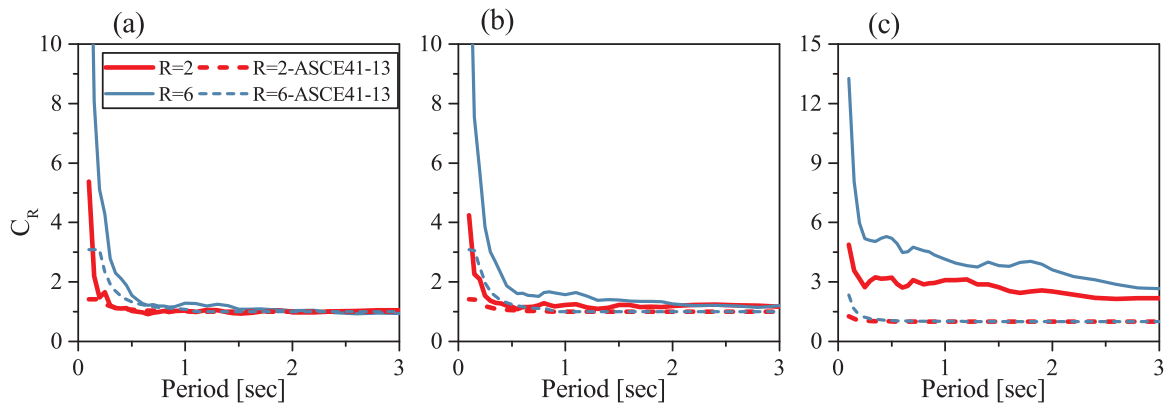


Fig. 11. Comparison of  $C_R$  with  $C_1$  coefficient in ASCE-41-13 for systems with bilinear hysteresis model: (a) fixed-base; (b)  $a_0 = 1$ ,  $h/r = 3$ ; and (c)  $a_0 = 3$ ,  $h/r = 5$ .

mentioned before, the effects of strength reduction factor on  $C_R$  values in soil-structure systems with MIB hysteresis behavior are not constant and can vary with the hysteresis status of structures.

Fig. 14b shows the effects of non-dimensional frequency on  $\gamma$  values for  $h/r = 3$  and  $R = 6$ . It can be seen that the values of  $\gamma$  increase with an increment of  $a_0$  for all type of hysteresis behaviors considered here. Fig. 13c illustrates the effects of aspect ratio ( $h/r$ ) on  $\gamma$  values for  $a_0 = 3$  and  $R = 4$ . It is observed that in short period region the value of  $\gamma$  decreases by increasing the aspect ratio. However, in long period region, slenderizing the structure will lead the  $\gamma$  be decreased. The results show that it is un-conservative to neglect the effects of SSI on inelastic displacement ratios. Generally, except for short period structures, soil-structure interaction brings down the maximum displacement of superstructure; however, the rate of reduction in inelastic structure is lower than the elastic one (Fig. 15). Therefore, inelastic displacement ratios of soil-structure systems are generally greater than fixed-base structures.

#### 6.4. Effect of hysteresis behavior

In order to study the effects of hysteresis behavior on inelastic displacement demand of soil-structure systems, the parameter  $\beta$  is defined as the ratio of inelastic displacement ratio in degraded structure to that in the non-degrading system:

$$\begin{aligned} \beta_{CL} &= \frac{C_{R,CL}}{C_{R,BL}}, \\ \beta_{SD} &= \frac{C_{R,SD}}{C_{R,BL}}, \\ \beta_{MIB} &= \frac{C_{R,MIB}}{C_{R,BL}} \end{aligned} \tag{11}$$

This ratio was computed for various values of fixed-base periods and strength reduction factors as well as for different hysteresis behavior subjected to the selected earthquake ground motions listed in Table 1. Bilinear hysteresis model (BL) is chosen as the baseline hysteresis model, because it does not exhibit degradation.

Mean of  $\beta$  values corresponding to hysteresis models that are considered herein, are shown in Fig. 16. It can be seen that the stiffness degradation has no significant effects on inelastic displacement ratios of soil-structure systems, except for short period structures (Fig. 16a). Degraded unloading stiffness (SD hysteresis behavior) extends the influence of stiffness degradation to a wider range of periods. As expected, strength-stiffness degradation (MIB hysteresis behavior) has significant effects especially for higher value of strength reduction factor.

As shown in Fig. 16b, the  $\beta$  values in the soil-structure systems with stiffness degrading is not significantly different from those observed in

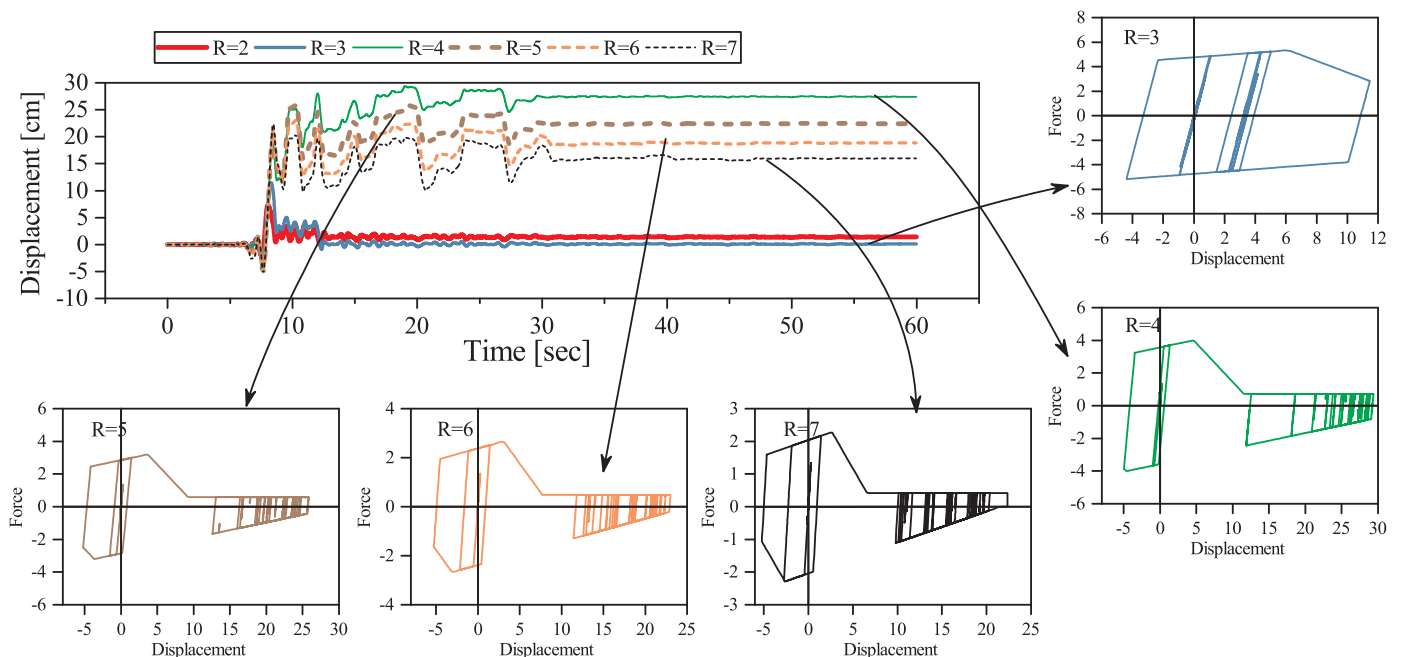


Fig. 12. Displacement time history and hysteretic response of soil-structure systems with MIB hysteresis model for earthquake ground motion RSN776,  $h/r = 1$ ,  $a_0 = 3$  and  $T_n = 0.45$  s.

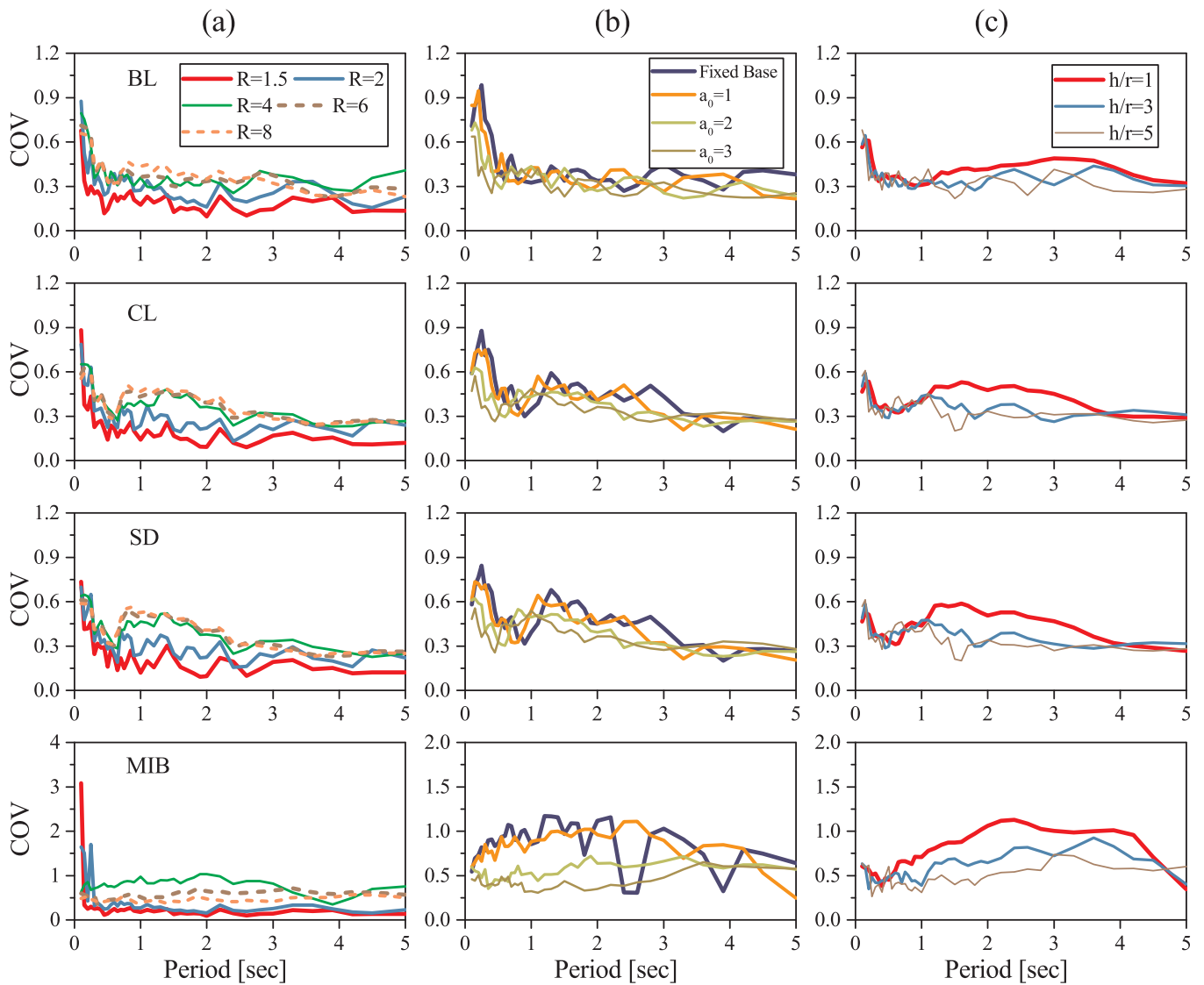


Fig. 13. Coefficient of variation of inelastic displacement ratios for: a)  $h/r = 3, a_0 = 2$ ; b)  $h/r = 3, R = 6$  and c)  $a_0 = 3, R = 4$ .

fixed-base structures. However, for MIB model, the value of  $\beta$  increases with an increment in  $a_0$ . Slenderizing the structures has no influence on the  $\beta$  values, except for MIB model in which the value of  $\beta$  increases as the aspect ratio ( $h/r$ ) raises (Fig. 16c).

In this part, further studies are carried out to determine the effects of hysteresis model and ductility capacity in modified Ibarra-Medina-Krawinkler model on inelastic displacement ratios. The investigated system parameters for this purpose are summarized as:

1. Ductility Capacity

- a) Special structures,  $\delta_c/\delta_y = 6$  (Very Ductile – MIB-S)
- b) Intermediate structures,  $\delta_c/\delta_y = 4$  (Medium Ductile – MIB-I)

2. Hysteresis model type

- a) Bilinear (MIB)
- b) Peak-oriented (MIP)

The term ductility defines as the ability of a structure to undergo inelastic deformation without losing the strength considerably. Fig. 17 shows the effects of ductility capacity in MIB hysteresis model on  $C_R$  values. It can be seen that the effects of ductility capacity on inelastic

displacement ratios are significant for higher values of strength reduction factor. As expected, any increase in ductility capacity will be accompanied by reduction in inelastic displacement of both fixed- and flexible-base systems.

To obtain a general idea of how different types of modified Ibarra-Medina-Krawinkler hysteresis models could affect inelastic displacement demand of soil-structures systems, Fig. 18 is provided for the mean inelastic displacement ratios of structures with both MIB and MIP hysteresis behaviors. The results indicate that except in short period range, inelastic displacement for MIP is smaller than MIB model. In short period range, the behavior is reversed and the inelastic displacement of structure of MIP hysteresis behavior is larger than that of the MIB model. Also, it is observed that the effects of peak-oriented model (MIP) are significant for the higher values of  $R$ .

7. Summary and conclusions

A comprehensive parametric study has been carried out to investigate the effects of SSI on inelastic displacement ratios ( $C_R$ ) of degraded structures. In this regard, inelastic displacement ratios are computed for soil-structure systems with four hysteretic models a) bilinear, b) modified Clough, c) stiffness-degrading, and d) strength-stiffness degrading, with different lateral strength level subjected to 19

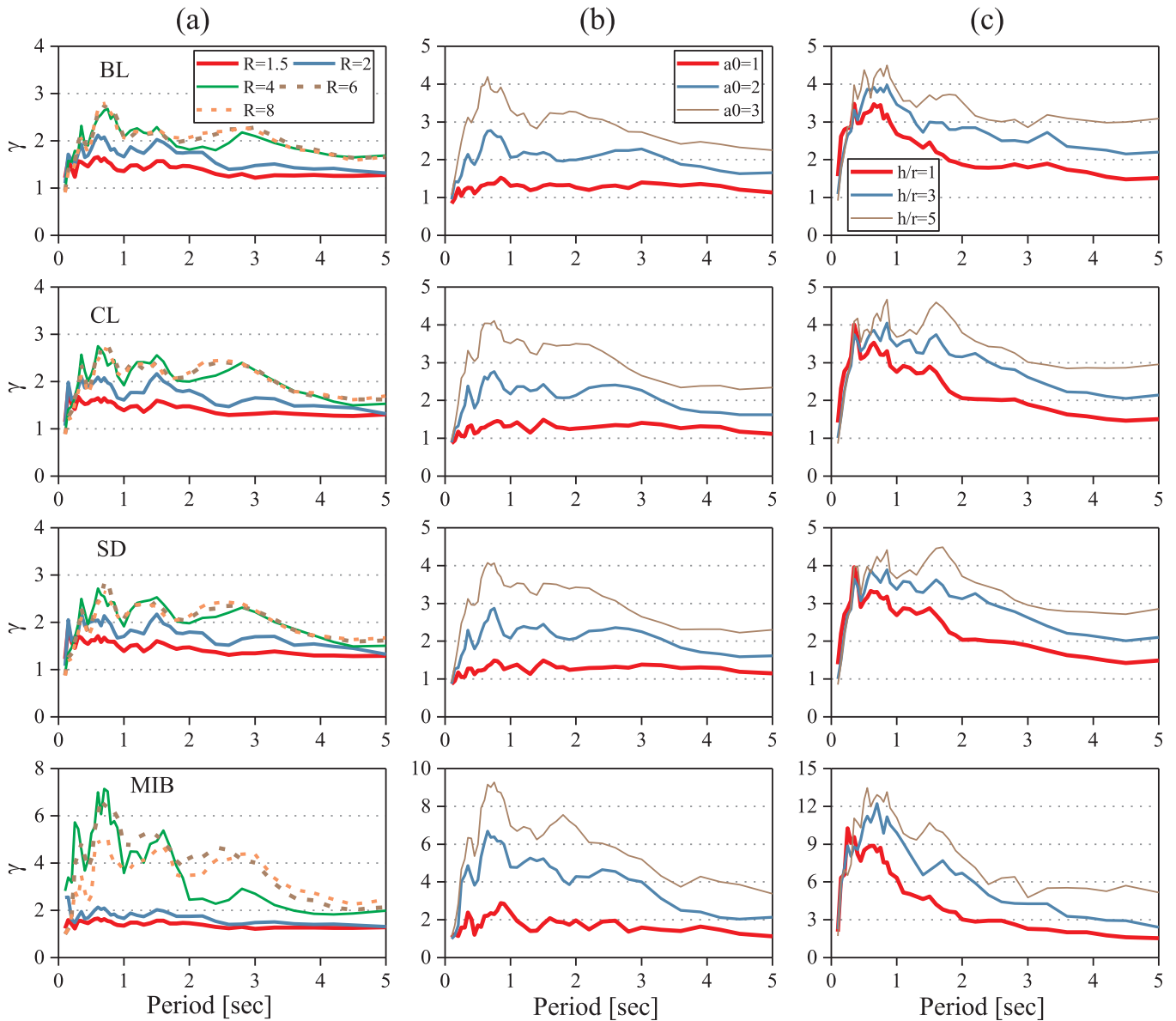


Fig. 14. Flexible to fixed-base inelastic displacement ratios for: a)  $h/r = 3, a_0 = 2$ ; b)  $h/r = 3, R = 6$  and c)  $a_0 = 3, R = 4$ .

strong ground motions. Influences of the SSI key parameters, hysteresis model and strength reduction level on inelastic displacement ratios are evaluated. In addition, a parametric investigation is performed to evaluate the parameters that could affect nonlinear response of structures with strength-stiffness degrading hysteretic model. The main conclusions are summarized as follows:

- For both fixed- and flexible-base structures, in short period region maximum inelastic displacements are greater than the elastic ones. However, for fixed-base structures the ratio of the maximum inelastic displacement to the maximum elastic displacement tends to reach unit value as the period of structure increase. This phenomenon is known as equal displacement rule which is not valid for soil-structure systems.

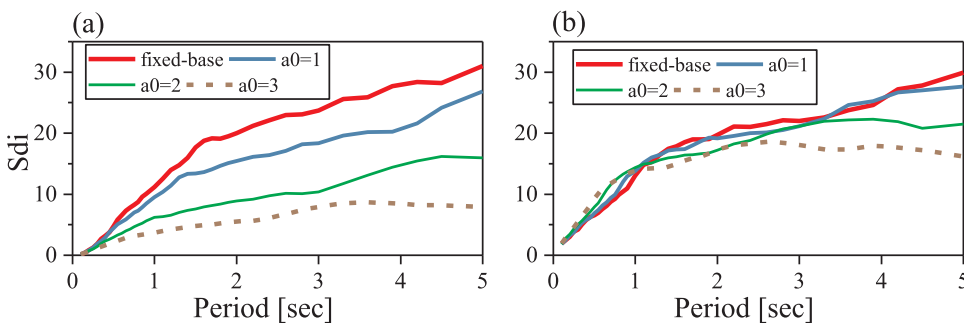


Fig. 15. Spectral displacements of soil-structure systems for  $h/r = 5$  and a)  $R = 1$ , b)  $R = 6$ .

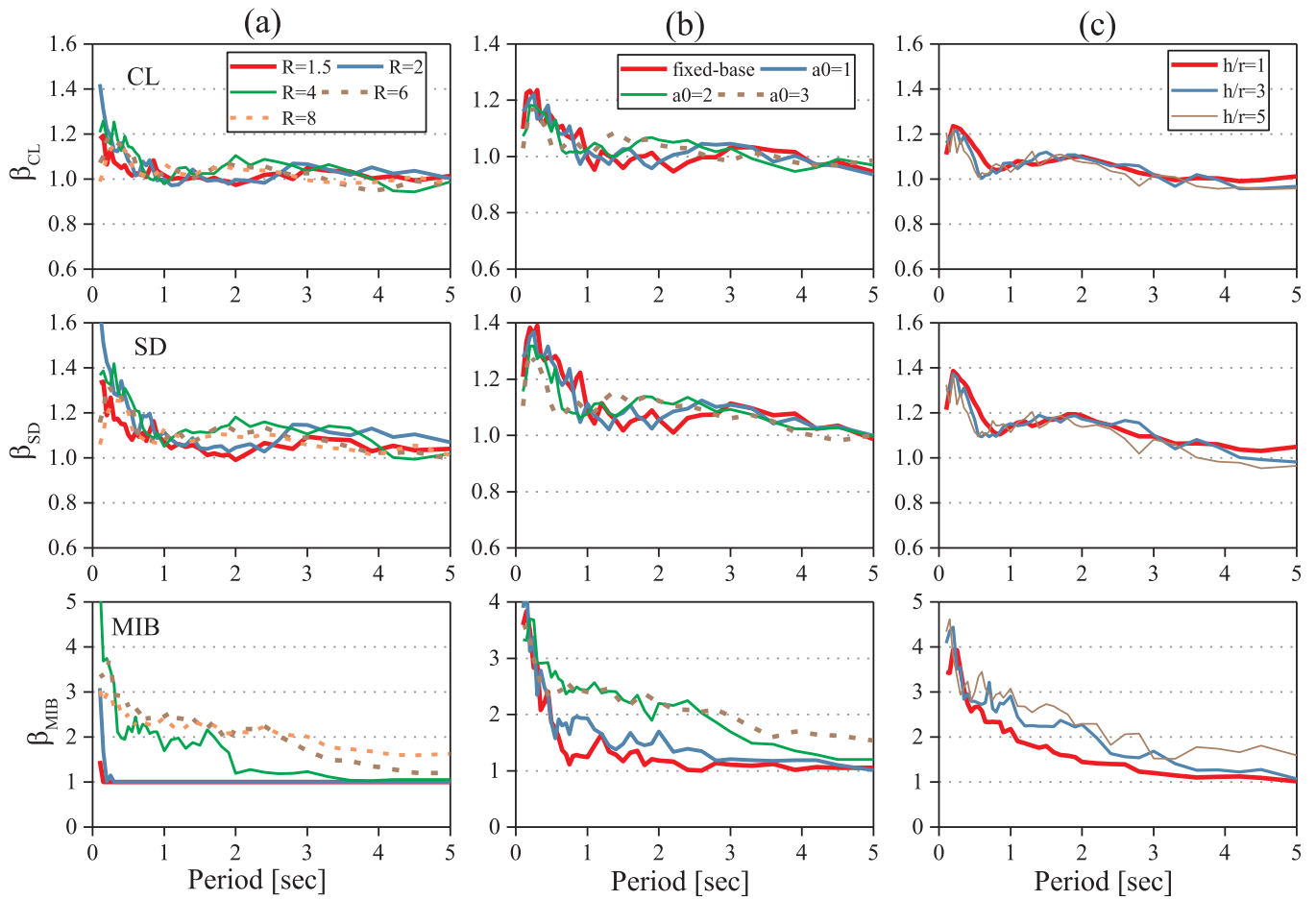


Fig. 16. Degrading to non-degrading inelastic displacement ratios computed for: a)  $h/r = 3, a_0 = 2$ ; b)  $h/r = 3, R = 6$  and c)  $a_0 = 3, R = 4$ .

- The results of this study are compared with coefficient  $C_1$  proposed by ASCE-41-13 [35]. It was observed that the  $C_1$  underestimates inelastic displacement ratios of soil-structure systems, especially when the soil-structure interaction effects are substantial.
- For flexible-base structures, in long period region the  $C_R$  values tend to reach a constant value. This constant value and the corresponding period range varies with non-dimensional frequency ( $a_0$ ) and aspect ratio ( $h/r$ ) values. Also, it is observed that increasing of the aspect ratio of long period structure increases the inelastic displacement ratio.
- For very short-period structures with bilinear and stiffness

degrading hysteretic models, coefficient of variation (COV) is not sensitive to the variations of the strength reduction factor, and generally takes larger values. However, for other ranges of period, coefficients of variation of inelastic displacement ratios increase by an increase in strength reduction factor, with the exception of strength-stiffness degrading models. Dispersion of  $C_R$  reduces as the non-dimensional frequency and aspect ratio increase.

- Generally, SSI increases the inelastic displacement ratios with exception of very short period structures. Also, increasing the strength reduction factor  $R$  leads to increase in the value of  $C_R$  for SSI systems with bilinear and stiffness degrading hysteretic models compare to

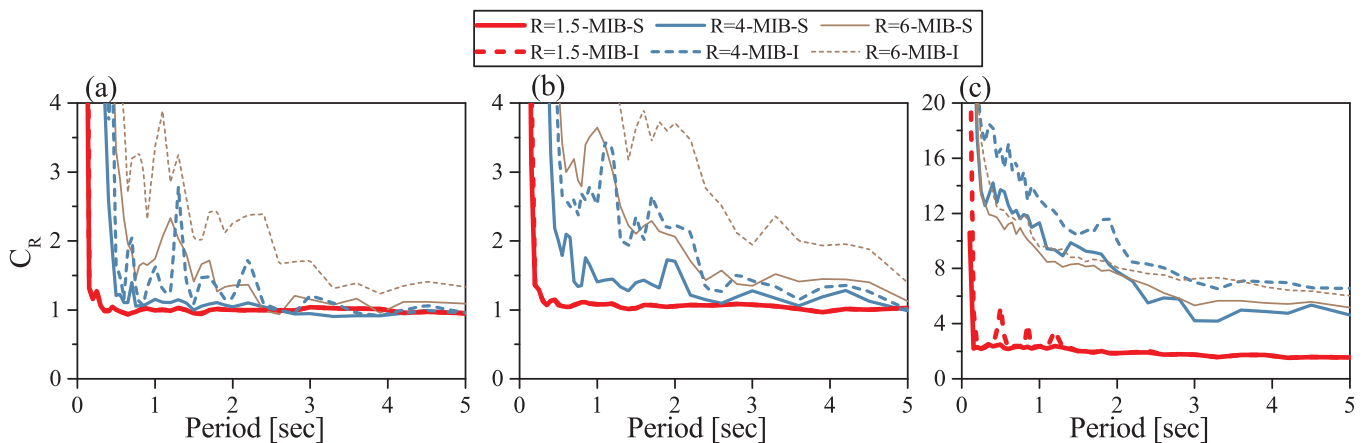


Fig. 17. Effects of ductility capacity in MIB hysteresis model on inelastic displacement ratios a) fixed-base b)  $h/r = 1, a_0 = 1$  and c)  $h/r = 3, a_0 = 3$ .

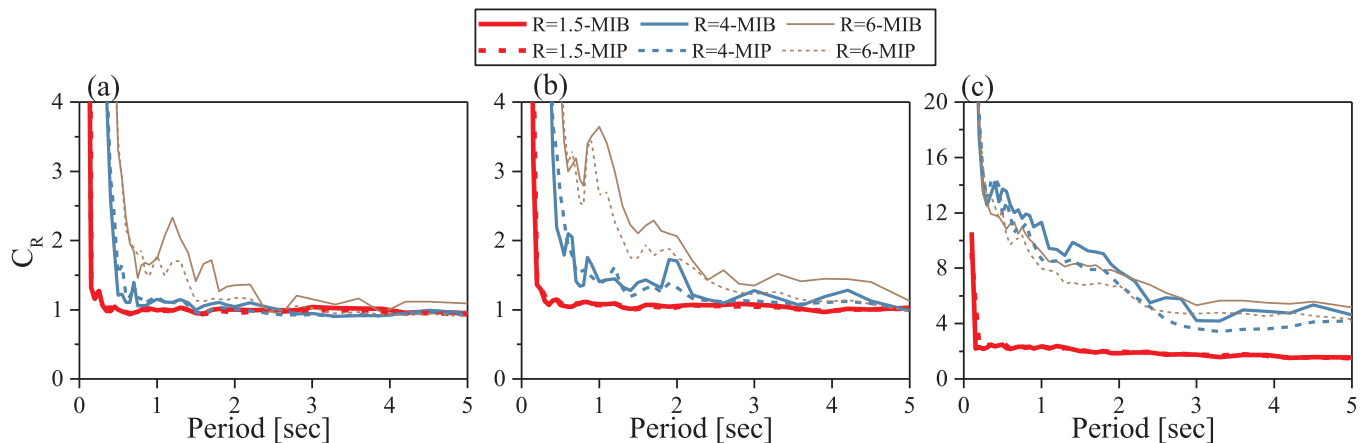


Fig. 18. Effects of modified Ibarra-medina-Krawinkler hysteresis models on inelastic displacement ratios: a) fixed-base b)  $h/r = 1$ ,  $a_0 = 1$  and c)  $h/r = 3$ ,  $a_0 = 3$ .

the fixed-base systems especially for the periods larger than 0.5 s. It is found that, regardless of hysteric model, the inelastic displacement ratios of soil-structure systems increase with an increment of  $a_0$ . Also, in long period region the  $C_R$  values increase with a growth in aspect ratio. But, in short period range the behavior is reversed and slenderizing the structures makes the  $C_R$  values decrease.

- In order to evaluate the effects of hysteresis behavior on  $C_R$  values of soil-structure systems, the ratio of inelastic displacement ratio in degrading structure to that in non-degrading system,  $\beta$  was computed. Results indicate that stiffness degradation can increase inelastic displacements of short period structures. For flexible-base structures, strength-stiffness degradation has significant effects and can increase inelastic displacement considerably. It is observed that, inelastic displacements of soil-structure systems with strength-stiffness degrading hysteresis model increases as the aspect ratio and  $a_0$  rise.
- The effects of ductility capacity in strength-stiffness degrading hysteresis model are substantial for higher values of strength reduction factor and, as expected, any increase in ductility capacity will be accompanied by a reduction in inelastic displacement demands of both fixed- and flexible-base systems.
- It is demonstrated that inelastic displacement demand of strength-stiffness degrading model with peak oriented behavior in long period region is smaller than that of the strength-stiffness degrading model with bilinear behavior. But in short period range the behavior is reversed.

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